2026.03.09 corrected solution to a problem from today

Monday, March 9, 2026 6:07 PM

$$\begin{cases} y'' + y' - 2y = 1 \\ y(0) = 0, \quad y'(0) = 4 \end{cases}$$

$$\mathcal{L}\{f''\}(s) = s^2F(s) - sf(0) - f'(0).$$

$$\mathcal{L}\{f'\}(s) = sF(s) - f(0).$$

Take $\mathcal{L}$ to get

$$\mathcal{L}\{y''\} + \mathcal{L}\{y'\} - 2\mathcal{L}\{y\} = \mathcal{L}\{1\}$$

$$\left[s^2Y(s) - s(\overset{y(0)}{0}) - 4\right] + \left[sY(s) - 0\right] - 2Y(s) = \frac{1}{s}$$

$$Y(s)(s^2+s-2) = \frac{1}{s} + 4$$

$$\boxed{Y(s) \overset{(*)}{=} \frac{1}{s(s^2+s-2)} + \frac{4}{s^2+s-2} = \frac{1}{s(s+2)(s-1)} + \frac{4}{(s+2)(s-1)}}$$

Partial fractions:

① $\dfrac{1}{s(s+2)(s-1)} = \dfrac{A}{s} + \dfrac{B}{s+2} + \dfrac{C}{s-1}$

$$1 = A(s+2)(s-1) + B(s)(s-1) + Cs(s+2)$$

$s=1$: $1 = 3C$, $C = \frac{1}{3}$

$s=-2$: $1 = 6B$, $B = \frac{1}{6}$

$s=0$: $1 = -2A$, $A = -\frac{1}{2}$

write as $s-(-2)$

$$\Rightarrow \frac{1}{s(s+2)(s-1)} = \frac{-1/2}{s} + \frac{1/6}{s+2} + \frac{1/3}{s-1}$$

② $\dfrac{4}{(s+2)(s-1)} = \dfrac{A}{s+2} + \dfrac{B}{s-1} \Longrightarrow 4 = A(s-1) + B(s+2)$

$s=1$: $4 = 3B$, $B = \frac{4}{3}$

$s=-2$: $4 = -3A$, $A = -\frac{4}{3}$

$$\Rightarrow \frac{1}{(s+2)(s-1)} = \frac{(-4/3)}{s+2} + \frac{(4/3)}{s-1}$$

$$\frac{1}{6} - \frac{4}{3} = \frac{1}{6} - \frac{8}{6} = -\frac{7}{6}$$

Returning to $(*)$ and using ①, ②, we see

$$Y(s) = \left[\frac{-1/2}{s} + \frac{1/6}{s+2} + \frac{1/3}{s-1}\right] + \left[\frac{-4/3}{s+2} + \frac{4/3}{s-1}\right]$$

$$= \frac{-1/2}{s} + \frac{-7/6}{s-(-2)} + \frac{5/3}{s-1}$$

Therefore, since by the table,

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1, \quad \mathcal{L}^{-1}\left\{\frac{1}{s-(-2)}\right\} = e^{-2t}, \text{ and } \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} = e^{t},$$

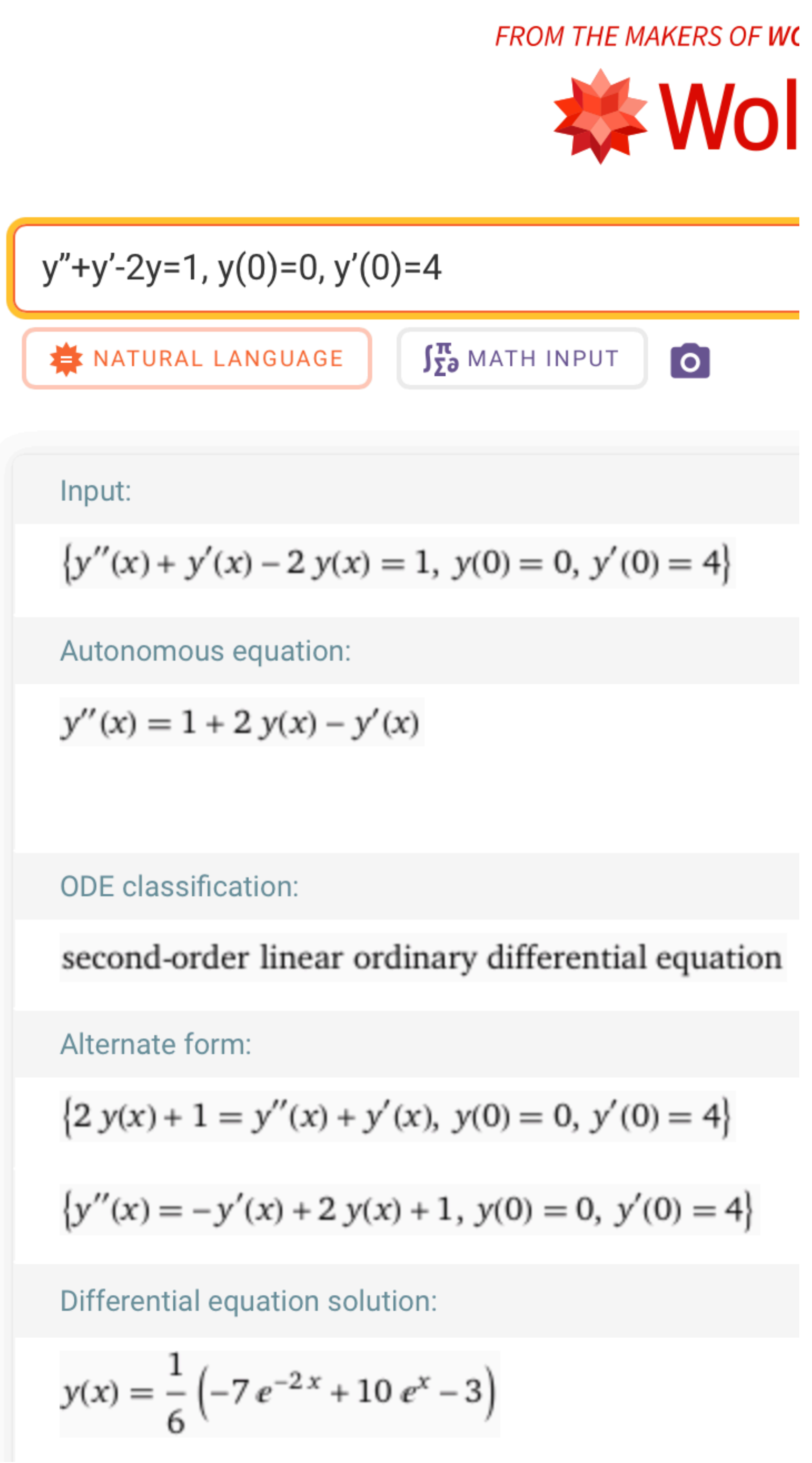
we now have shown that

$$y(t) = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{7}{6}\mathcal{L}^{-1}\left\{\frac{1}{s-(-2)}\right\} + \frac{5}{3}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$= -\frac{1}{2} - \frac{7}{6}e^{-2t} + \frac{5}{3}e^{-t}$$

(this solution is correct!:

https://www.wolframalpha.com/input?i=y''%2By'-2y%3D1%2C+y%280%29%3D0%2C+y'%280%29%3D4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y''+y'-2y=1, y(0)=0, y'(0)=4

NATURAL LANGUAGE | MATH INPUT

Input:

$\{y''(x) + y'(x) - 2\,y(x) = 1,\ y(0) = 0,\ y'(0) = 4\}$

Autonomous equation:

$y''(x) = 1 + 2\,y(x) - y'(x)$

ODE classification:

second-order linear ordinary differential equation

Alternate form:

$\{2\,y(x) + 1 = y''(x) + y'(x),\ y(0) = 0,\ y'(0) = 4\}$

$\{y''(x) = -y'(x) + 2\,y(x) + 1,\ y(0) = 0,\ y'(0) = 4\}$

Differential equation solution:

$y(x) = \frac{1}{6}\left(-7\,e^{-2\,x} + 10\,e^{x} - 3\right)$

$$= -\frac{7}{6}e^{-2x} + \underset{\frac{5}{3}}{\underset{\uparrow}{\frac{10}{6}}}e^{x} - \underset{\frac{1}{2}}{\underset{\uparrow}{\frac{3}{6}}}$$